# CS 516—Software Foundations via Formal Languages—Spring 2025

# Problem Set 7

### Due by 11:59pm on Thursday, May 1

### Problem 1 (35 points)

Define a function diff  $\in \{0,1\}^* \to \mathbb{Z}$  by: for all  $w \in \{0,1\}^*$ ,

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diff w = the number of 1's in w - 2 (the number of 0's in w).
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Thus

- diff % = 0;
- diff 0 = -2;
- diff 1 = 1;
- for all  $x, y \in \{0, 1\}^*$ ,  $\operatorname{diff}(xy) = \operatorname{diff} x + \operatorname{diff} y$ .

Let  $Y = \{ w \in \{0,1\}^* \mid \text{for all prefixes } v \text{ of } w, \text{diff } v \ge 0 \}$ . Find a grammar G such that L(G) = Y, and prove that your answer is correct.

#### Problem 2 (35 points)

Let the language X be

$$\{0^i 1^j 2^k 3^l \mid i, j, k, l \in \mathbb{N} \text{ and } i+j=k+l \text{ and}$$
  
*i* is even and *j* is odd and *k* is even and *l* is odd  $\}$ .

Use Forlan to help find a grammar G such that L(G) = X.

- Try to do as much as possible of the work of finding G using Forlan.
- But as a secondary goal, attempt to minimize the number of variables of G, and for grammars with an equal number of variables, the number of productions of G.

#### Problem 3 (30 points)

Let the language X be

$$\{ 0^i 1^j 2^k \mid i, j, k \in \mathbb{N} \text{ and } i < j < k \}.$$

Prove that X is not context-free.