**CS 516—Software Foundations via Formal Languages—Spring 2025**

# Final Examination

**Tuesday, May 6, 12noon-2pm**

**Question 1 (20 points)**

Suppose we know that $i, i', j, j', k, k' \in \mathbb{N}$ and

$$\mathsf{0}^i\mathsf{1}^j\mathsf{0}^k = \mathsf{0}^{i'}\mathsf{1}^{j'}\mathsf{0}^{k'}.$$

What can we conclude about the relationship between $i, j, k$ and $i', j', k'$? (You don't need to prove that your answer is correct.)

**Question 2 (20 points)**

Let $X = \{\, \mathsf{0}^i\mathsf{1}^j\mathsf{2}^k \mid i, j, k \in \mathbb{N} \text{ and } 0 < i + j < k \,\}$. Find a grammar $G$ such that $L(G) = X$.

**Question 3 (20 points)**

Given $w \in \{\mathsf{0}, \mathsf{1}\}^*$, we write:

- $\mathbf{zeros}\, w$ for the number of occurrences of $\mathsf{0}$ in $w$; and
- $\mathbf{ones}\, w$ for the number of occurrences of $\mathsf{1}$ in $w$.

Define the following languages:

$$\begin{aligned}
X_{\mathsf{ee}} &= \{\, w \in \{\mathsf{0}, \mathsf{1}\}^* \mid \mathbf{zeros}\, w \text{ is even and } \mathbf{ones}\, w \text{ is even} \,\}, \\
X_{\mathsf{eo}} &= \{\, w \in \{\mathsf{0}, \mathsf{1}\}^* \mid \mathbf{zeros}\, w \text{ is even and } \mathbf{ones}\, w \text{ is odd} \,\}, \\
X_{\mathsf{oe}} &= \{\, w \in \{\mathsf{0}, \mathsf{1}\}^* \mid \mathbf{zeros}\, w \text{ is odd and } \mathbf{ones}\, w \text{ is even} \,\}, \\
X_{\mathsf{oo}} &= \{\, w \in \{\mathsf{0}, \mathsf{1}\}^* \mid \mathbf{zeros}\, w \text{ is odd and } \mathbf{ones}\, w \text{ is odd} \,\}.
\end{aligned}$$

Assume that we have already proven the following facts: (1) $\% \in X_{\mathsf{ee}}$; (2) $X_{\mathsf{ee}}\{\mathsf{0}\} \subseteq X_{\mathsf{oe}}$; (3) $X_{\mathsf{ee}}\{\mathsf{1}\} \subseteq X_{\mathsf{eo}}$; (4) $X_{\mathsf{eo}}\{\mathsf{0}\} \subseteq X_{\mathsf{oo}}$; (5) $X_{\mathsf{eo}}\{\mathsf{1}\} \subseteq X_{\mathsf{ee}}$; (6) $X_{\mathsf{oe}}\{\mathsf{0}\} \subseteq X_{\mathsf{ee}}$; (7) $X_{\mathsf{oe}}\{\mathsf{1}\} \subseteq X_{\mathsf{oo}}$; (8) $X_{\mathsf{oo}}\{\mathsf{0}\} \subseteq X_{\mathsf{eo}}$; and (9) $X_{\mathsf{oo}}\{\mathsf{1}\} \subseteq X_{\mathsf{oe}}$.

Find a DFA $M$ such that $L(M) = X_{\mathsf{eo}} \cup X_{\mathsf{oe}}$, and prove that your answer is correct, explicitly making use of the above facts.

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**Question 4 (20 points)**

Let the language $X$ be

$$\{\, \mathsf{0}^i\mathsf{1}^j\mathsf{2}^k\mathsf{3}^l \mid i,j,k,l \in \mathbb{N} \text{ and } i+j=k+l \text{ and } \\ i \text{ is even and } j \text{ is odd and } k \text{ is even and } l \text{ is odd} \,\}.$$

Prove that $X$ is not regular.

**Question 5 (20 points)**

Suppose $\alpha$ and $\beta$ are regular expressions whose alphabets are subsets of $\{\mathsf{0},\mathsf{1}\}$. Let

$$X = \{\, w \in \{\mathsf{0},\mathsf{1}\}^* \mid \text{for all } x,y \in \{\mathsf{0},\mathsf{1}\}^*, \text{ if } w = xy, \text{ then, if } x \in L(\alpha), \text{ then } y \notin L(\beta) \,\}.$$

Explain how we can use the algorithms we've studied to create a regular expression $\gamma$ such that $L(\gamma) = X$. (You don't need to worry about making $\gamma$ small.)